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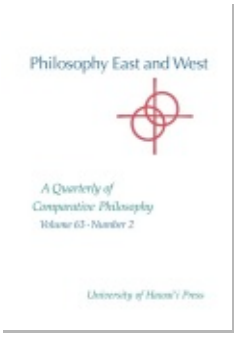

THE STRUCTURE OF EMPTINESS

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Introduction

The view that everything is empty *(śūnya)* is a central metaphysical plank of Mahāyāna Buddhism. This is understood in different ways in different Mahāyāna traditions. Our concern here will be with how it is understood in the Mādhyamaka school of Nāgārjuna and his commentators, such as Candrakīrti and Tsongkhapa.[1] This notion of emptiness has often been the focus of objections. Perhaps the most important of these is to the effect that it entails a nihilism: nothing exists. The objection, in turn, is denied by Mādhyamaka theorists, such as Nāgārjuna. One of the things that makes the debate difficult is that the precise import of the view that everything is empty is unclear. The point of this essay is to put the debate in a new light. I will do this by offering a mathematical characterization of Emptiness—that is, the totality of empty things[2]—showing that, whatever it is, it has a precise structure and is not, therefore, to be identified with nothingness.

Before turning to the mathematization, however, I will provide an informal look at the matters at hand. Even in canonical expositions of the emptiness of things, the idea is often explained in different, and not obviously equivalent, ways. A central part of its meaning is, however, that nothing has intrinsic existence (self-existence, *svabhāva*). In terms of Western philosophy, we might think of this as saying that there are no ultimate substances, that is, things that would exist even if everything else went out of existence. Nothing exists in and of itself. Everything that exists does so inasmuch as, and only inasmuch as, it relates to other things. It has, so to say, only relational existence.[3] But what is this?

Relational Existence in Western Philosophy

To explain the notion of relational existence more clearly, especially for philosophers who may know little of Buddhist philosophy, let us start by looking at an example from Western philosophy. This concerns space and time. Famously, Newton was an absolutist about space and time. He held that

> absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration.

And that

> absolute space, in its own nature, without relation to anything else, remains always similar and immovable.[4]

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Newton held that spatial and temporal locations exist in themselves, and would exist even if there were no physical things that occupied space and time.

By contrast, Leibniz was a relationalist about space and time. He held

> space to be merely relative, as time is; that . . . it . . . be an order of coexistences, as time is an order of successions.[5]

In other words, spatial and temporal locations have no intrinsic existence. Physical events bear temporal relations (befores and afters) to each other, and there is nothing more to occurring at a certain time than having certain of these relationships to other things. Similarly, physical objects have spatial relations to each other (norths and souths), and there is nothing more to being in a certain place than having these relations to other things.

Newton and Leibniz each gave various arguments for their views, but we will not be concerned here with what they were, or who was right in the matter. I have stated their positions only to illustrate the notion of relational existence. For Newton, spatial and temporal locations exist in and of themselves: they have self-existence. By contrast, for Leibniz, they have no self-existence. A spatial/temporal position is simply a locus in a field of spatial/temporal relations. That is, it has only a relational existence.

That various kinds of entity have no absolute, but only a relational, existence is a not uncommon view in Western philosophy. Of other examples that could be given, let me cite just one.[6] Words and sentences have meanings. What is the status of meanings? Many philosophers and linguists have held that there are intrinsically existing meanings, entities that exist over and above words and sentences. A notable person who held this view was, of course, Frege. For him, the senses of words and sentences exist in just this way. By contrast, structuralist linguists, beginning with Saussure, hold that meanings do not have self-existence. Words and sentences enter into various kinds of relationships with other words and sentences. For example, 'blue' contrasts with 'red' in a way that 'scarlet' does not. To have a certain meaning is simply to be related to other words/sentences in certain ways. That is, meanings have no intrinsic existence, merely a relational one.

Examples such as these illustrate, I hope, the notion of self-existence versus relational existence. To have self-existence is to exist independently of anything else. By contrast, things that have relational existence are simply loci in a field of relations, and are individuated by that location.

Relational Existence in Mādhyamaka

One of the foundational claims of early Buddhist psychology is that there is no self. This does not mean that there is no Graham Priest (for example), currently wandering around in Melbourne. Rather, it means that, on closer inspection, Graham Priest unravels into a bunch of parts (aggregates, *skandas*) that come together at a certain time, interact, change, and finally fall apart. Though it is often quite useful to think of this dynamic bunch as a single thing and give the name 'Graham Priest' to it, there is

no thing, no entity, that answers to this name. That is, Graham Priest is empty of self-being *(svabhāva).*

As a result of this doctrine, and the pervasiveness of change and impermanence in the Buddhist perspective on things, early Buddhists spent much time analyzing the parts of the self and other things, and investigating the interactions between these parts. However, in this Abhidharma tradition, it was held that such analysis always grounds out into ultimates that do have self-existence, *dharma*s. Hence arose a distinction between the conventional existence *(saṃvṛti-satya)* of the kind that Graham Priest has and the ultimate existence *(paramārtha-satya)* of the kind the *dharmas* have.

With the rise of Mahāyāna Buddhism, the extension of the realm of conventional existents was widened. The notion of conventional existence itself also came in for further scrutiny. Thus, according to Nāgārjuna, the founder of Mādhyamaka, to exist (conventionally) is to exist only in relation to other things (which may be parts, but may be other things as well). Thus, the seer and the seen exist only in relation to one another,[7] and the agent and the action exist only in relation to one another.[8] More generally, consider anything that comes into existence and goes out of existence, say you. What is it for *you* to exist? What makes an existent *you*? What defines your identity is that you were born of certain parents at a certain time, have a certain DNA, went to a certain school, had certain friends, were affected by the things you saw and did, and so on. You are simply the locus of all these causal interactions: there is no self-existent entity that occupies that locus. This argument is adumbrated by Nāgārjuna in the first chapter of the *Mūlamadhyamakakārikā (MMK),* and articulated at much greater length by Candrakīrti in his *Mādhyamakāvatāra* and elsewhere.

The argument is not completely general, however, since it applies only to contingent things in the world; it does not apply, for example, to space and time. Nāgārjuna considers time in chapter 19 of the *MMK.* He attacks the idea of times as self-existent and argues that time exists only relationally. For example (*MMK* XIX.3):

> If [the present and the future] are not dependent on the past,
> Neither of the two would be established.
> Therefore neither the present
> Nor the future would exist.

As Garfield puts it in his commentary on this chapter (*The Fundamental Wisdom of the Middle Way,* p. 254):

> Nāgārjuna … argues that time cannot be conceived of as an entity existing independently of temporal phenomena, but must itself be regarded as a set of relations among them.

Leibniz, of course, would have agreed.[9]

Śūnyatā

So much for the notion of relational existence in both Western and Mādhyamaka philosophy. Now to *śūnyatā.* The Mādhyamaka widened the extension of emptiness

so much that it was held to apply to *everything.* There are no dharmas; everything is empty; everything disappears under ultimate analysis and has only relational existence. *Śūnyatā,* in the sense in which we are going to understand it here, is precisely this doctrine. As far as I know, no Western philosopher has endorsed such a radical view.[10]

The canonical defense of the view is to be found in the *MMK.* In this, Nāgārjuna goes through all the things that one might think to have self-existence, and argues that they do not. Many of the arguments employed concern the kind of thing in question, such as matter, time, and consciousness. But some of the arguments are quite general. Here is one such argument from chapter 5 (or at least, my interpretation of it; interpreting Nāgārjuna is always a sensitive issue).[11]

Take an object that one might suppose to have self-existence. Since the argument is quite general, *anything* will do, but for the sake of illustration, let us take Australia. Australia has various properties (characteristics): it has a certain current geographical location, it was colonized by Europeans in the eighteenth and nineteenth centuries, it is called 'Australia,' and so on. Now, to be Australia just is to be the bearer of these characteristics. (So any entity which bore—related to—those properties would be Australia.) Australia, then, does not have self-existence: to be (identical to) Australia is to be related to these properties in this way.

One might balk at some of the details of this argument. To be Australia, it may not be necessary to have *exactly* those characteristics that Australia has, maybe just some of them, perhaps just some essential ones, or perhaps just enough to constitute a family resemblance, or whatever. Here we touch upon the details of certain Western theories of identity. The details are not important, however. In all these theories, to be Australia is to be related to a certain bunch of properties in a certain way. In other words, Australia has relational existence.

One way to attempt to avoid this conclusion is to suppose that there is something to being Australia that is independent of the possession of any characteristics. Maybe Australia is the substance which *bears* all the properties. Different substances (or bits of substance?) might bear exactly the same properties; still, one might be Australia and the other might not be. But on this account, to be Australia is to be a certain substance, *to be that very substance.*[12] Australia's existence is certainly not, therefore, independent of the bearing of properties. On this account, to be Australia is exactly to be the bearer of the italicized characteristic.[13]

There are certainly more things to be said about this argument, but we need not go into any of this here. I give it only to illustrate what sorts of considerations might drive one to the conclusion that everything is empty of self-existence, that existence, in the sense that anything can have it, is itself a 'relative characteristic.'[14]

Nihilism

Let us now move on to the claim that *śūnyatā* entails nihilism. In his *Mūlamadhyamakakārikā,* Nāgārjuna was very much concerned to reject this charge. It is not clear whom, precisely, he conceives to be making it, or what, exactly, their arguments

were. But such a thought will come naturally to any Buddhist working in the older Abhidarma tradition. As I have already noted, in slightly different words, the tradition insists on a fundamental distinction between primary existents *(dravyasat)* and secondary existents *(prañaptisat),* with the latter being conceptually constructed out of the former. For an Abhidharmika, then, the Mādhyamaka doctrine of universal emptiness implies the apparent absurdity that everything is conceptually constructed, though there is nothing out of which to construct it.

A natural way of spelling this thought out is as a regress argument. Take any object *a;* consider the things out of which it is constructed (if there are any); now consider the things out of which *they* are constructed; and so on. This process must ground out at some stage, or *a* would be made out of nothing, so it would be a no-thing. I know of no Buddhist text that spells out the argument in exactly this way. But regress arguments of this kind for the existence of atomic substances were certainly used by the Hindu Vaiśeṣika philosophers,[15] whom Nāgārjuna would also have had in mind.[16]

Let me spell out the regress argument more carefully. Suppose that the doctrine of emptiness is correct. Then, for any object *a, a* has no intrinsic existence; it is merely a locus in a field of relations, $R_0, \ldots, R_n$. But what of these relations? If the doctrine is right, these have no intrinsic existence either: each relation, R_i, is simply a locus in another field of relations, $R_{i_0}, \ldots, R_{i_m}$. And what of each of these relations, R_{i_j}? Each of them has no intrinsic existence, but is simply a locus in another field of relations, et cetera. Clearly, we have an infinite regress,[17] and the regress may well be taken to be vicious, in the following sense. The existence of any thing is constituted by, and only by, the existence of other things, whose existence is constituted by, and only by, the existence of other things, and so on. Since there is nothing that grounds this process, there is nothing that ultimately constitutes the existence of anything. Nothing, therefore, exists. Emptiness entails nihilism.

This argument is also of a kind that is familiar in Western philosophy. Let me give another couple of instances of it to try to render its cogency clearer. The first is from Kant's *Critique of Pure Reason.* In the Second Antinomy, Kant gives arguments for the claim that matter must both be and not be infinitely divisible. In his own words, the *reductio* against infinite divisibility goes as follows:

> Let us assume that composite substances are not made up of simple parts. If all composition then be removed in thought, no composite part, and (since we admit no simple parts) also no simple parts, that is to say, nothing at all, will remain, and accordingly, no substance will be given. (A434 = B462)[18]

In other words, consider any substance: suppose that it is a composition of smaller parts, and each such part is a composition of smaller parts, and each such part is a composition of smaller parts, and so on *ad infinitum.* Then, upon complete decomposition, there is nothing left. Thus, there could have been no substance there in the first place.[19]

The second example comes from Wittgenstein's *Tractatus,* and is to the effect that the world must contain what he calls 'substances.' These are not physical sim-

ples as in Kant, however, but logical simples; that is, things which ultimately ground the meanings of propositions. In his own words, the argument goes as follows:

> If the world had no substance, then whether or not a proposition had a sense would depend on whether another proposition was true.
>
> In that case we could not sketch any picture of the world (true or false). (2.0211–0212)[20]

The argument is a dark one. But what I take it to mean is something like this. Suppose a proposition had meaning by virtue of its decomposition into meaningful parts, and that each of these was meaningful by virtue of its decomposition into meaningful parts, and so on *ad infinitum.* Then there would be nothing, in the end, to determine the meaning of anything. No proposition, then, would have determinate sense.[21]

All three of these arguments just considered have the same structure:

> If the *X* of a thing *a* were constituted by *a*'s relationship to other things, whose *X* were constituted by their relationship to other things, whose *X* were constituted by their relationships to other things, and so on indefinitely, then there would be nothing to ultimately determine the *X* of *a; a* would therefore have no *X.*

In the three arguments, *X* is existence, substance, and meaning, respectively. In each case, the *X* in question disappears under ultimate analysis. In each case, we analyze and analyze—until nothing is left.

At this point one might object. The regress does not show that there is nothing left—just nothing *determinate.* We can simply accept that the kinds of thing in question (meaning, substance, existence) are, perhaps surprisingly, indeterminate kinds of things. But this does not show that they do not exist. The *Vimalakīrti-nirdeśa Sūtra,* for example, explicitly advises one to learn to "tolerate the groundlessness of things."[22]

It is here that the debate becomes murky. Can existence be indeterminate? What would it be to exist indeterminately? Surely, it might be thought, if something exists, it is exactly what it is. How could there be any indeterminacy about this? But if something is in a continuous transition from being red to being blue, it might be thought, there are times when it is indeterminate as to whether or not it is red. Similarly, if something can come into and go out of existence by degrees, couldn't there be times when it is indeterminate as to whether or not it exists? Wouldn't this be indeterminate existence? The answers to all these questions are opaque. Doubtless there is much good philosophy to be done in addressing them. However, I want now to take the issue off in a direction that allows more precision.

Relational Existence Again

Often the precise import of an idea can be made clearer if the view in question can be mathematized. Thus it is that mathematical analyses have been deployed over and over again in Western physics and twentieth-century metaphysics. It is time to turn this technique on emptiness. Let us start by returning to the question of rela-

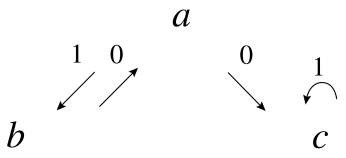

Figure 1

tional existence. To be empty is to exist only as the locus in a field of relations. How can one understand this mathematically?[23]

A natural way of thinking about objects and the relations between them is as a labeled graph. Thus, suppose that we have a set of objects, X_0 (why the subscript we will see in due course), and a bunch of relations, $\rho = \{R_i : i \in I\}$, between them. Suppose, for simplicity, that $X_0 = \{a, b, c\}$ and that $\rho = \{R_0, R_1\}$. Then the graph might be depicted in the diagram in figure 1. Arrows denote the relations; superscripts to the arrows denote the relation in question. This conception takes the objects as independently existing. How could we understand them as simply loci in the network of relations? Suppose that one simply has the relations. Each R_i has a collection of instances, $\langle R_i \rangle$. The standard way to think of these is as sets of ordered pairs—or, actually, ordered triples, the third member indexing the relation. Thus, in the example above, the members of $\langle R_0 \rangle$ would be $\langle b, a, 0 \rangle$ and $\langle a, c, 0 \rangle$, and those of $\langle R_1 \rangle$ would be $\langle a, b, 1 \rangle$ and $\langle c, c, 1 \rangle$. This, of course, is already to think of the objects in X_0 as existing in themselves. Since we do not wish to make this assumption, we will not adopt this reductive analysis of instances, but think of them, instead, as *sui generis.* However, bearing the reduction in mind will provide a useful heuristic in what follows.

We may now consider a certain relation R_L that holds between instances. Intuitively, the relation holds between those instances that originate from the same object. Thus, if one again thinks of instances as ordered sets, $\langle x, y, i \rangle\ R_L\ \langle z, w, j \rangle$ iff x is z, but if one is to jettison this idea, R_L must be understood as a primitive relation, and, as the heuristic indicates, it is natural to take this to be an equivalence relation.

One can now identify loci in a very simple matter. A locus is determined by a non-empty bunch of instances that bear the R_L relation to each other. One can, if one likes, think of this as a class. If α is any relation-instance, it determines a locus $\{\beta : \alpha R_L \beta\}$. The loci are simply equivalence classes of instances.[24]

This is not quite right yet. Strictly speaking, this captures the loci only of those objects that are in the domain (left-hand argument) of some R_i. In general, some objects may be in only the co-domain (right-hand argument) of all R_i, and some may be isolated points, and so in neither the domain nor the co-domain of any R_i. Thus, consider the diagram in figure 2. Neither b nor c is in the domain of a relation, and d is in neither the domain nor the co-domain of any relation.

A simple way to solve these problems is to insist that ρ be closed under complementation. If R is any relation, its complement, $\bar{R}$, is the relation such that $x\bar{R}y$ iff it is not the case that xRy. If there is no y such that xRy, it is not the case that xRx, and so $x\bar{R}x$. So if ρ is closed under complementation, x is a left-hand argument of some

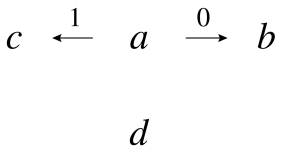

Figure 2

relation in ρ. In the present context, the requirement that ρ be closed under complementation is a very natural one. If R is a relation on a bunch of objects, $\bar{R}$ is another natural and obvious relation on them, obtained simply by employing 'not.' Thus, if R is *'x loves y,'* $\bar{R}$ is simply *'x does not love y.'*

With this restriction in place, the construction does indeed capture all loci in a graph. Specifically, and again thinking of objects as independently existing for a moment, we can map X_0 into the set of loci by the function that maps x to $\hat{x} = \{\alpha : \text{for some } y \in X_0 \text{ and } i \in I,\ \alpha = \langle x, y, i\rangle\}$.[25] This is a one-to-one correspondence. Moreover, any relation R on the domain of objects induces a corresponding relation $R^{\uparrow}$ on the loci. Namely, $\hat{x}R^{\uparrow}\hat{y}$ iff xRy. The map $\hat{}$ is then an isomorphism.[26]

As an aside, I note that we may treat not just the objects but also the relations between them, as themselves sets of instances of a certain kind. R_i may be identified with $\hat{R}_i = \{\alpha : \text{for some } x \in X_0, \text{ and } y \in X_0,\ \alpha = \langle x, y, i\rangle\}$.[27] If we do this, then the relations on loci may be defined in terms of the relations between the instances, plus set-membership. Thus, let R_P be the permutation relation on instances such that, intuitively, $\langle x, y, i\rangle\ R_P\ \langle y, x, i\rangle$. Then, as is not difficult to check: $\hat{x}R^{\uparrow}\hat{y}$ iff xRy iff for some $\alpha \in \hat{x}$ and $\beta \in \hat{y}$ ($\alpha \in \hat{R}$ and $\alpha R_P \beta$).

What we see, then, is that we may dispense with objects and the relationships between them, and operate equivalently in terms of loci and the relationships between these. The ontology of independent objects may be replaced by an ontology of loci.

Emptiness and Nihilism

So far so good. Now let us come to the doctrine that everything is empty. This says that everything has only relational existence. We have just seen how to interpret the claim that the members of X_0 have only relational existence: they are to be understood as loci. Loci are sets of relation-instances. Let the set of things of this kind (sets of relation-instances) be X_1. Then we have seen how to trade in an ontology of the objects in X_0 (with relations between them) for an ontology of the objects in X_1 (with relations between them). Now the members of X_1 must themselves be taken to be empty. But since they have a structure of the same kind as that with which we started, we can simply repeat the analysis, to obtain objects of a new kind X_2 (with relations between them). But this, again, has the same structure, and is to be analyzed in the same way. We repeat the process to the limit (ω times). The result might well be thought to be nihilism. At each stage, a certain ontology is thrown away and replaced by another. In the limit, every ontology has been thrown away.

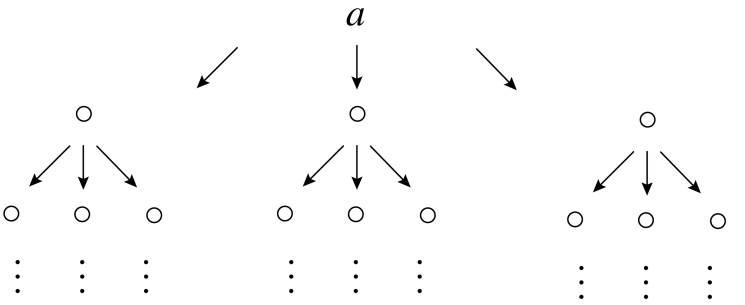

Figure 3

But the appearance is deceptive. As we have just seen, each X_i can be embedded in (that is, is isomorphic to) a part of X_{i+1}. If we identify the isomorphic objects, we obtain a chain of sets $X_0 \subseteq X_1 \subseteq X_2 \subseteq \ldots$. At each stage, what we obtain, therefore, is a richer structure. The result at the limit is $\bigcup_{i<\omega} X_i = X_\omega$. This is how things turn out, in fact, to be when we apply relentlessly the thought that everything is empty; and X_ω is not the empty set.[28]

An even more interesting perspective emerges if we interpret loci as sets of relation-instances, but now subject—not the sets—but the relation-instances themselves, to the same kind of recursive analysis. This takes the relation-instances to be sets of relation-instances (of a higher order). We then iterate the process. To see what happens when we do this, take an object a in X_0 as an example. This is a class of instances. Let us suppose, for the sake of illustration, that there are only three instances, b_0, b_1, and b_2. Thus, $a = \{b_0, b_1, b_2\}$. Each b in turn is a class. Suppose, again for the sake of simplicity, that each b_i is $\{c_{i_1}, c_{i_2}, c_{i_3}\}$. Then $a = \{\{c_{0_0}, c_{0_1}, c_{0_2}\}, \{c_{1_0}, c_{1_1}, c_{1_2}\}, \{c_{2_0}, c_{2_1}, c_{2_2}\}\}$. And so on. If we pursue this to the limit, we obtain a non-well-founded set that can be depicted in the diagram in figure 3. In orthodox set theory (*ZF* with the Axiom of Foundation) there are no non-well-founded sets of this kind. But there are perfectly respectable set theories where there are such sets.[29] In such a set theory, membership regresses *may* bottom-out to give a perfectly well-founded set. Thus, consider the well-founded set $a = \{\alpha, \{\beta, \gamma\}, \{\delta, \varepsilon, \eta\}\}$, where the Greek letters now represent non-sets (or the empty set). This is represented by the diagram in figure 4.

Some chains may bottom-out, while others do not. Such a situation might be as shown in the diagram in figure 5. Alternatively, no branch may ever bottom out. Let us call sets where this is the case *purely non-well-founded sets*.[30] When the analysis of relational existence is pursued to its limit, this is the sort of set to which it gives rise. There is never a ground to the regress. As we noted, the *Vimalakīrti-nirdeśa Sūtra* tells us to learn to "tolerate the groundlessness of things."[31] In the case of a purely non-well-founded set, the set has no determinate content: it is pure form. As the *Heart Sūtra,* the most famous of all Mahāyāna sūtras puts it: form is empty; emptiness is form.[32]

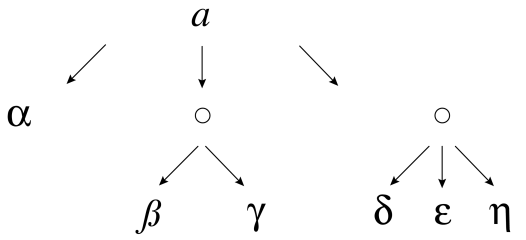

Figure 4

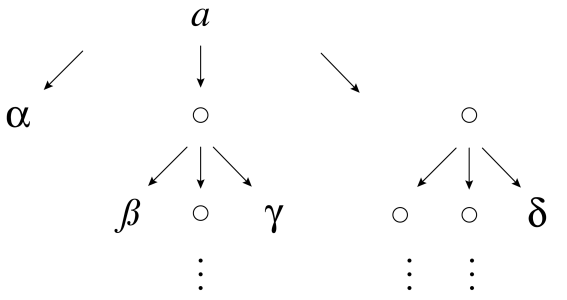

Figure 5

It might be suggested that the analysis here is still incomplete. We are still left with an ontology of purely non-well-founded sets, with the relations between them, especially set-membership. Indeed, as we saw, we may take $\in$ (and its negation, $\bar{\in}$) to be the only such relation. These would seem to have self-existence (non-relational existence). Should we not apply the analysis to these, too? In fact, there is no need. As far as the analysis went, the initial set of objects, X_0, could have been anything. There is no problem about supposing that it contained all purely non-well-founded sets in the first place! This gives rise to a new element of non-well-foundedness. For consider the relation-instances that hold between objects in X_0. These are relation-instances between sets, and the sets are sets of relation-instances between objects of X_0 (and similarly for X_i, $i > 0$). But non-well-foundedness of this kind is obviously no problem in the present context.

It is worth noting that there are other instances of non-well-foundedness in the offing. The Mādhyamaka insist that when it is said that everything is empty, this includes Emptiness itself. Emptiness, in the sense that we are interpreting the notion, is just the totality of things, X_0. Thus, to say that emptiness is empty can be thought of as saying that $X_0 \in X_0$. This gives rise to a regress of the form: $X_0 \ni X_0 \ni \ldots$.

At any rate, and to return to the question with which we started, what is to be said about nihilism from this perspective? Reality is the totality of all objects, X_0, Emptiness. This, being a set of purely non-well-founded sets, is itself a purely non-well-founded set. Again, we see that Emptiness is not to be identified as the empty set (which is a well-founded set). As the Mādhyamaka insist, Emptiness is not a nothing: it has a determinate structure, one of pure form.

Conclusion

I am well aware that defending certain Buddhist views by appealing to modern set theory is anachronistic. I do not think that this is a problem. The fact that the appropriate mathematics was not around when the canon was formulated does not mean that it cannot be applied to it. Although non-well-founded set theory was a conceptual exercise to which the writers of the canon had no access, they did their best without it, as we have seen in talking of the *Vimalakīrti-nirdeśa Sūtra.* Perhaps that part of the canon that comes closest to the spot is the discussion of Emptiness in the *Avataṃsaka* (Flower Garland) *Sūtra.*[33] This has the famous image of the Net of Indra. Reality is a net at each node of which is a bright jewel. Each jewel reflects each other jewel—reflecting each other jewel, reflecting each other jewel.... This is perhaps as striking an image of non-well-foundedness as it is possible to get in non-mathematical terms. Though the image has a good deal more poetic charm than the mathematics, I have no doubt that the writers of the sūtra could only have approved.

Notes

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1 – The literature on emptiness is enormous. For an introduction, see P. Williams, "The Buddhist Concept of Emptiness," in E. Craig, ed., *The Routledge Encyclopedia of Philosophy* (London: Routledge, 1998), vol. 2, pp. 75–80. See also the essays in part 1 of J. Garfield, *Empty Words* (Oxford: Oxford University Press, 2001), and D. Lopez, *Elaborations on Emptiness* (Princeton, NJ: Princeton University Press, 1998).

2 – The word 'emptiness' is sometimes used for the property of being empty as well, but this will not be our concern here.

3 – Another part of its meaning is that nothing has, in the jargon of Western philosophy, primary properties. All properties are *secundum quid.* Indeed, the claim about existence can be thought of as a special case of this: even existence is a relational property.

4 – Isaac Newton, "Absolute Space and Time," in J.J.C. Smart, ed., *Problems of Space and Time* (London: Macmillan, 1964), pp. 81–88.

5 – Gottfried Leibniz, "The Relational Theory of Space and Time," in Smart, *Problems of Space and Time,* pp. 89–98.

6 – There are certainly others. One is Marx's view of the commodity. A commodity is not a thing in itself, but just something that occupies a relational role (notably between producer and consumer) in certain capitalist practices (see *Capital,* vol. 1, chap. 1, sec. 4). Another is the view of certain structuralists in the philosophy of mathematics, who hold that a mathematical object is simply a locus in a structure (see G. Hellman, "Three Varieties of Mathematical Structuralism," *Philosophia Mathematica* 9 [2001]: 184–211).

7 – "Without detachment from vision, there is no seer./ Nor is there a seer detached from it./ If there is no seer/ How can there be seeing or the seen" (*Mūlamadhyamakakārikā* (hereafter, *MMK*), III:6; all translations are from J. Garfield, *The Fundamental Wisdom of the Middle Way: Nāgārjuna's Mūlamadhyamakakārikā* [Oxford: Oxford University Press, 2001]).

8 – "Action depends upon the agent./ The agent itself depends upon action./ One cannot see any way/ to establish them differently" (*MMK,* VIII:12).

9 – The agreement may not be a coincidence. Leibniz was familiar with parts of Chinese Philosophy, including Hua Yen Buddhism, a school of Buddhism much influenced by Mādhyamaka. See F. Perkins, *Leibniz and China* (Cambridge: Cambridge University Press, 2004).

10 – Though maybe some structuralists have come close to it. For example, "that the world is made up of relationships rather than things, constitutes the first principle of the way of thinking that can properly be called 'structuralist'" (T. Hawkes, *Structuralism and Semiotics* [London: Methuen, 1977], pp. 17–18).

11 – An anonymous referee objected to this interpretation of Nāgārjuna on the grounds that I am attributing to him, implausibly, the Frege/Russell descriptive theory of names. This is certainly incorrect: the argument does not even mention names; it is about objects.

12 – Such individual essences are often termed by their medieval name: *haecceities.*

13 – And just as the object could not exist without its characterizations, they, in turn, could not exist without it. As Nāgārjuna says, when he summarizes the corresponding argument at stanza 27 of the *Seventy Stanzas*:

> Without depending on the defined [i.e., the characterized object] one cannot establish the definition [i.e., the characterizations] and without considering the definition one cannot establish the defined. As they depend on each other, they have not arisen by themselves, so therefore the defined and the definition are devoid of inherent existence.... (D. R. Komito, *Nāgārjuna's "Seventy Stanzas": a Buddhist Psychology of Emptiness* [Ithaca, NY: Snow Lion Publications, 1987], p. 85)

14 – Garfield, *The Fundamental Wisdom of the Middle Way,* p. 151.

15 – See J. Koller, *Asian Philosophies,* 4th ed. (Upper Saddle River, NJ: Prentice-Hall, 2002), p. 74. Koller also claims (chap. 14) that Sarvāstivāda Buddhists used regress arguments of this kind.

16 – An anonymous referee noted that there was another popular argument for the view that the regress must ground out. Unless the regress comes to an end, everything would be composed of an infinite number of things, so everything would have the same size, namely, infinite. Deploying the tools of modern measure theory, one can answer this objection too, though going into this here would take us too far away from present concerns.

17 – The regress may go round in a loop at some stage, but a regress that repeats itself is still a regress.

18 – Translation is taken from N. Kemp Smith, trans., *Immanuel Kant's Critique of Pure Reason,* 2nd ed. (London: Macmillan, 1933).

19 – A similar argument is used by Aristotle, *On Generation and Corruption,* 316a15–34.

20 – Translation taken from D. F. Pears and B. F. McGuinness, trans., *Tractatus Logico-Philosophicus* (London: Routledge and Kegan Paul, 1961).

21 – See sect. 12.6 of G. Priest, *Beyond the Limits of Thought* (Cambridge: Cambridge University Press, 1995; 2nd ed., Oxford: Oxford University Press, 2002).

22 – R. A. Thurman, *The Holy Teachings of Vimilakīrti: A Māhāyana Scripture* (Philadelphia, PA: Pennsylvania State University Press, 1976), p. 73.

23 – The way I pursue in what follows may well not be the only way. For example, it might be possible to employ category theory or some of the other technical vehicles employed in mathematical structuralism to provide a different sort of analysis. The analysis I give, however, strikes me as simple and direct.

24 – J. Keränen ("The Identity Problem for Realist Structuralism," *Philosophia Mathematica* 9 [2001], pp. 308–330) poses a problem for certain kinds of mathematical structuralism, concerning how to give the identity conditions for the loci of structures. The identity conditions of loci given here constitute neither of the kinds Keränen considers, and so avoid the objection. Identity is defined in terms of relation-instances and the relations between them. It is true that these are taken to be self-existent entities, so the account has something in common with Keränen's haecceity account. As we shall see in the next section, however, this is only a temporary measure.

25 – Alternatively, and in standard fashion, we may take $\hat{x}$ to be a some particular member of this set.

26 – *Proof*: A locus is a non-empty set of the form $\{\alpha : \exists y \in X_0, \exists i \in I, \alpha = \langle x, y, i \rangle\}$. Since $\hat{}$ maps x to this set, the function is onto. If $\{\alpha : \exists y \in X_0, \exists i \in I, \alpha = \langle x, y, i \rangle\} = \{\alpha : \exists y \in X_0, \exists i \in I, \alpha = \langle z, y, i \rangle\}$, then $x = z$, so the function is one to one. Finally, the definition of $\uparrow$ makes $\hat{}$ structure-preserving by definition.

27 – As may be clear, the relation-instances are just tropes, and this construction is of a fairly standard kind in trope theory. See, for example, J. Bacon, *Universals and Property Instances: The Alphabet of Being* (Oxford: Blackwell, 1995).

Trope theory is not unknown in classical India. For example, the metaphysical theory of the fifth-century Buddhist logician Diṅnāga can be seen as a trope theory. See J. Ganeri, *Philosophy in Classical India* (London: Routledge, 2001), esp. chap. 4. There is also an important connection between trope theory and category theory. On this, see J. Bacon, "Tropes," *Stanford Encyclopedia of Philosophy,* http://plato.stanford.edu/entries/tropes/, 2002.

28 – Since isomorphic elements may not be literally identical to each other, perhaps a better way of looking at the matter is as follows. At each stage of the proceedings an ontology is thrown away and replaced by another with a richer structure. When all the ontologies have been thrown away, all that is left is the structure of the limit itself, X_ω. This is not the empty set. So Emptiness is not the same as nothing.

29 – See the systems in P. Aczel, *Non-Well-Founded Sets,* CSLI Lecture Notes (Chicago, IL: University of Chicago Press, 1988).

30 – One might wonder how to account for the identity and difference of such non-well-founded sets. Several different answers are possible (see Aczel, *Non-Well-Founded Sets*). One is to the effect that two such sets are identical if they have the same graph. Thus, $\{\{\{\ldots\}\}\}$ is distinct from $\{\{\{\ldots\}\{\ldots\}\}\{\{\ldots\}\{\ldots\}\}\}$ since the graphs of these sets are, respectively, as shown in the diagram in figure 6.

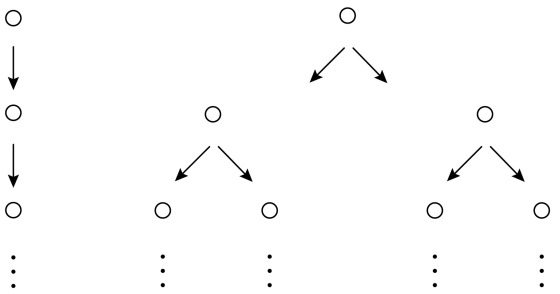

Figure 6

31 – Perhaps we all have some psychological resistance to infinite regresses (such, for example, is the appeal of Christian Cosmological arguments for the existence of God). This might well dispose us to take the regresses involved here to bottom out, and so to reify bottom-most elements. It is a standard Buddhist psychology that reification is involved in the false and *duḥkha*-generating worldview that needs to be seen through.

32 – D. Lopez, *Elaborations on Emptiness* (Princeton, NJ: Princeton University Press, 1998).

33 – T. Cleary, trans., *The Flower Ornament Scripture: A Translation of the Avatamsaka Sutra* (Boulder, CO: Shambhala Publications, 1984).