Mark Siderits and J. Dervin O'Brien

Zeno and Nāgārjuna on motion

Similarities and differences between Zeno's Paradoxes and Nāgārjuna's arguments against motion in Chapter II of *Mūla-mādhyamika-kārika* (MMK II) have already been remarked by numerous scholars of Indian philosophy. Thus for instance Kajiyama refers to certain of Nāgārjuna's arguments as "Zeno-like,"[1] and Murti seeks to show that Nāgārjuna's dialectic is innately superior to Zeno's.[2] In both cases the assumption is made that Zeno's arguments are specious; the authors seek to dissociate Nāgārjuna's destructive dialectic from the taint of the best-known piece of destructive dialectic in the Western tradition. On Brumbaugh's analysis of the four Paradoxes, however, Zeno's arguments are seen to form a coherent whole which, *as a whole*, constitutes a valid argument against a certain type of natural philosophy (valid, that is, so long as one does not accept Cantorian talk of "higher-order infinities"). The target of the Paradoxes is now seen as Pythagorean atomism, with its curious—and to the modern mind incompatible—mixture of the principles of continuity and discontinuity as applied to the analysis of space and time. Zeno's genius lies in separating out of this muddle the four possible permutations of spatio-temporal analysis, and then constructing a paradox to show the implausibility of each account. Only on this interpretation of the Paradoxes can we account for the renown which they enjoyed in the ancient world.[3]

As we shall see, however, the atomisms of ancient India were strikingly similar in several respects to the doctrines of Pythagoreanism. This and the clear correspondence of at least one of Nāgārjuna's arguments against motion to one of Zeno's Paradoxes, lead us to wonder whether a new look at the relationship between the two philosophers might not be in order. In particular, we wonder whether, armed with the insight into atomistic doctrines and their refutation which Brumbaugh's analysis affords, we might be able to give a more plausible interpretation of at least some of Nāgārjuna's arguments than has hitherto been possible. There is no question but that Zeno and Nāgārjuna put their respective refutations of motion to completely different uses. The question is whether the two employ similar strategies. On our understanding of the Paradoxes a sympathetic account of Nāgārjuna is no longer in danger of "contamination" from specious Eleatic reasoning. Thus the principal aim of the following will be to exhibit what seem to us to be some striking parallels between certain of Zeno's and Nāgārjuna's arguments, both in methodologies and in targets.

Eleatic philosophy, of which Parmenides was the principal exponent and Zeno the staunch defender, was in part an attack on Pythagorean science, which explained the world in terms of a multiplicity of opposing principles. The Eleatics maintained that Being was fundamentally one and unchanging—and therefore, of course, immovable. Such a counterintuitive position required exceptionally strong arguments to support it, the best of which were supplied

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by Zeno. The rigor of his arguments overwhelmed his contemporaries, and the most famous of these arguments, the Paradoxes, continues to fascinate laymen and philosophers alike.

Many attempts have been made to explain these Paradoxes. Taken as separate and independent arguments, they range from the peculiar to the silly; yet in the ancient world they enjoyed an enormous reputation. The best resolution of this problem is that offered by Robert Brumbaugh in *The Philosophers of Greece*: The Paradoxes should be viewed, not as separate arguments, but as four parts of a single argument, each part designed to refute one possible interpretation of Pythagorean philosophy of nature.

Because for many years the Pythagorean order imposed a rigid code of secrecy upon its members, it is impossible to determine with any certainty precisely what its official doctrine was at any given time. However it seems fair to say that Pythagorean science was basically atomistic, the universe being conceived of as additive, that is, composed of atoms or minims, indivisible "smallest-possible" units of space and time. This conception must have been dealt a severe blow by the Pythagorean discovery that the hypotenuse of a unit right triangle was incommensurable with its sides, and that therefore there could be *no* one unit, however small, of which both could be composed. Attempts to resolve this difficulty led to great ambiguity as to the nature of atoms, which varied according to context from entities of definite magnitude to dimensionless points and instants. The Pythagoreans maintained both that the world was composed of atoms and that any magnitude was infinitely divisible. No one definition of the atom would suffice. If it were taken to have definite magnitude, then there would be lines which could not be bisected, and no magnitude would be infinitely divisible; if, on the other hand, the atom were made dimensionless to give infinite divisibility, no quantity of such atoms could ever add up to any magnitude at all. According to Brumbaugh, Zeno's Paradoxes were designed to bring out the inherent absurdities of such a world view and to show that, however one interpreted this position, whichever of its premises one adopted, no account of motion could be given which did not end in absurdity. Whether space and time were atomistic or infinitely divisible, no intelligible account of motion through them was possible.

There are four possible combinations here: Space might be continuous (that is, infinitely divisible) and time discrete (that is, composed of extended minims or atoms); or space might be discrete and time continuous; or both might be continuous; or, again, both might be discrete. The Bisection Paradox, Achilles and the Tortoise, the Arrow, and the Stadium are designed to refute, respectively, each of these possibilities. Each Paradox depends for its effect upon its proper suppressed premise concerning the nature of space and time.

The Bisection Paradox assumes that space is continuous (infinitely divisible) and time discrete (atomistic). Zeno presents it as follows:

Aristotle *Phys.* Z9, 239b11 ... *πρῶτος μὲν ὁ περὶ τοῦ μὴ κινεῖσθαι διὰ τὸ πρότερον εἰς τὸ ἥμισυ δεῖν ἀφικέσθαι τὸ φερόμενον ἢ πρὸς τὸ τέλος....*

... The first asserts the non-existence of motion on the ground that that which is in locomotion must arrive at the half-way stage before it arrives at the goal...[4]

The problem here is that the walker is required to traverse an infinite series of distances, which is impossible. Since time is discrete, in order to traverse each of the distances involved, the walker requires at least one minim of time. Therefore the journey requires an infinite number of such minims of time, that is, an infinite duration, and for this reason it can never be completed.

The paradox of Achilles and the Tortoise assumes that space is discrete and time continuous. It goes as follows:

Aristotle *Phys.* Z9, 239b14 *δεύτερος δ' ὁ καλούμενος 'Αχιλλεύς. ἔστι δ' οὗτος ὅτι τὸ βραδύτατον οὐδέποτε καταληφθήσεται θέον ὑπὸ τοῦ ταχίστου· ἔμπροσθεν γὰρ ἀναγκαῖον ἐλθεῖν τὸ διῶκον ὅθεν ὥρμησε τὸ φεῦγον, ὥστ' ἀεί τι προέχειν ἀναγκαῖον τὸ βραδύτερον. ἔστι δὲ καὶ οὗτος ὁ αὐτὸς λόγος τῷ διχοτομεῖν, διαφέρει δ' ἐν τῷ διαιρεῖν μὴ δίχα τὸ προσλαμβανόμενον μέγεθος.*

The second is the so-called Achilles, and it amounts to this, that in a race the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead. This argument is the same in principle as that which depends on bisection, though it differs from it in that the spaces with which we successively have to deal are not divided into halves.[5]

In this case, the difficulty arises from the fact that there is an infinite series of moments in which the tortoise is running. In each moment, the tortoise must traverse at least one minim of space. In order to overtake the tortoise, Achilles must traverse each spatial minim through which the tortoise has passed. Therefore, Achilles would have to travel an infinite distance in order to catch the tortoise. Like the Bisection Paradox, this problem can be simply stated thus: one can never complete an infinite series.

The Arrow Paradox, on the other hand, assumes both space and time to be continuous.

Aristotle *Phys.* Z9, 239b30 *τρίτος δ' ὁ νῦν ῥηθείς, ὅτι ἡ ὀιστὸς φερομένη ἕστηκεν. συμβαίνει δὲ παρὰ τὸ λαμβάνειν τὸν χρόνον συγκεῖσθαι ἐκ τῶν νῦν· μὴ διδομένου γὰρ τούτου οὐκ ἔσται ὁ συλλογισμός.* (Cf. *ibid.* 239b5, where, however, the text is corrupt.)

The third is that already given above, to the effect that the flying arrow is at rest, which result follows from the assumption that time is composed of moments: if this assumption is not granted, the conclusion will not follow.[6]

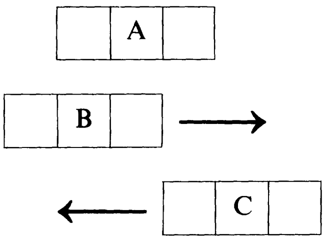
Because time is infinitely divisible, and because moments thus have no duration, at any given moment the arrow is standing still in a space equal to its length. Therefore, it is at every moment at rest, and thus it never moves. Once again, the problem can be simply stated: one cannot add a number of dimensionless instants together to achieve a duration; no matter how many such instants are added together, their sum will always be zero.

The paradox of the Stadium is for the modern reader the most baffling of the four, and our interpretation, which follows, differs from Brumbaugh's. We agree with him, however, that this puzzle assumes both space and time to be discrete—composed of minims.

Aristotle *Phys.* Z9, 239b33 *τέταρτος δ' ὁ περὶ τῶν ἐν σταδίῳ κινουμένων ἐξ ἐναντίας ἴσων ὄγκων παρ' ἴσους, τῶν μὲν ἀπὸ τέλους τοῦ σταδίου τῶν δ' ἀπὸ μέσου, ἴσῳ τάχει, ἐν ᾧ συμβαίνειν οἴεται ἴσον εἶναι χρόνον τῷ διπλασίῳ τὸν ἥμισυν.*

The fourth argument is that concerning the two rows of bodies, each row being composed of an equal number of bodies of equal size, passing each other on a race-course as they proceed with equal velocity in opposite directions, the one row originally occupying the space between the goal and the middle point of the course and the other that between the middle point and the starting-post. This, he thinks, involves the conclusion that half a given time is equal to double that time.[7]

Assume for the moment that we are speaking, as Zeno originally did, of bodies rather than chariots. Assume a stationary body (A) divided into three sections, each section being one minim long. Assume two more such bodies, one (B) traveling past (A) from left to right at a certain velocity, the other (C) traveling past (A) in the opposite direction at the same speed.



Let (B) be passing (A) at a velocity of one minim of space per minim of time. Then in the time (one temporal minim) in which the front edge of (B) passes one minim of (A), the front edge of (C) will pass two minims of (B), and in doing so the front edge of (C) will pass *one* minim of (B) in *half* a minim of time, which is impossible, since the minim is by definition indivisible.

This puzzle would work just as well looked at in another way. If we say that the second moving body (B) is passing the first (A) at the slowest possible speed, that is, one minim of space per minim of time, then in the same duration in which the front edge of (B) passes one minim of (C), it (B) will pass only *half* a minim of (A), the stationary body, which is impossible, since, once again, the minim is indivisible. Either way the key to understanding this Paradox lies in understanding that Zeno is not here assuming the atomicity of empty space or empty time; these concepts were foreign to the ancient Greeks, who thought instead in terms of the space-which-something-occupies, or the time-in-which-something-occurs. What is assumed here is, for example, the atomicity of the space-which-something-occupies, and therefore the atomicity of that which occupies the space, as well.

The interpretation of this Paradox turns on the phrase, "half a given time is equal to double that time." It should be borne in mind that the wording here is Aristotle's, not Zeno's, and that Aristotle clearly misunderstands this Paradox. He thinks that Zeno reasons fallaciously that a given object traveling at a given speed will pass two identical objects, one stationary and one itself in motion, in the same amount of time. Modern exponents of this same interpretation express it differently: Zeno, they say, is misled by his ignorance of the concept of relative velocity. Whichever way the alleged fallacy is stated, Zeno is not foolish enough to have committed it. He is not saying that (B) will pass (A) (stationary) and (C) (moving at the same speed as (B) but in the opposite direction) in the same amount of time; instead he is pointing out that, if (B) is traveling at, for example, a speed of one minim of space per minim of time, it will pass one minim of (A) in one minim of time, but it will pass one minim of (C) in half a minim of time, thus dividing the indivisible minim, which is impossible. The issue of relative velocity is irrelevant and anachronous.

Not one of these Paradoxes is, by itself, a convincing argument against motion, but each, when taken to include its proper assumptions about the nature of space and time, neatly disposes of one possible account of the universe in which motion occurs. (Of course, some of these arguments would serve for more than one case, but it is reasonable to assume that four were included for the sake of elegance.) Once the Paradoxes are seen as a destructive tetralemna, they then form an impressive demonstration that any additive conception of the universe renders an intelligible account of motion impossible.

Furthermore, these puzzles then can be seen as part of a comprehensive Eleatic argument against the possibility of motion. Fundamental to Eleatic philosophy is the premise that what is unintelligible cannot exist. Therefore, in order to demonstrate the impossibility of motion, one need only show that no matter what kind of universe one assumes, no intelligible account of motion can be given. It will then follow that motion cannot occur in any possible universe.

We begin with the assumption that the universe must be either additive (that is, made up of parts) or continuous (that is, made up, not of parts, but of a continuous, unbroken substance). If it is additive, then there are three possibilities: (1) that the universe is composed of bodies separated by a void; or, (2) that the universe is composed of minims; or (3) that the universe is composed of dimensionless points and instants. Case (1) is disposed of by Parmenides himself; he argues that the void is unintelligible, and therefore cannot exist, thus rendering (1) impossible. All possible permutations of (2) and (3) are refuted by Zeno's Paradoxes; no conceivable assortment of minims and dimensionless points and instants makes possible an intelligible account of motion. Thus, on Eleatic terms, the universe cannot be additive.

On the other hand, if the universe is continuous, then motion can only be explained in terms of compression and rarifaction. However, these are clearly species of change, and Parmenides argues that change of any kind is impossible, since it involves coming-to-be (that is, arising from nothing, which "nothing," since it is unintelligible, cannot exist) and passing-out-of-being (which requires that something which exists commence to not-exist, which is likewise unintelligible and therefore impossible). These arguments, it should be noted, all turn on the confusion of not-being (for example, being not-red) with nonbeing (nonexistence). However, if we accept them, as Zeno apparently did, then they do show that in a continuous universe, motion is impossible.

Thus, on Eleatic terms, no matter what kind of universe we suppose—continuous or additive—no intelligible account of motion can be given, and therefore motion is impossible. Although this and other of their conclusions never achieved wide acceptance, their arguments had enormous influence, establishing the rationalist tradition in philosophy which survives until today.

Before we proceed to a direct examination of Nāgārjuna's arguments against motion, we should like to say a few words about the historical background behind the writing of the *Mūla-mādhyamika-kārika* (MMK), with particular reference to Indian notions of space and time. While far less is known about ancient Indian mathematics and physics than is known about their ancient Greek counterparts, it is still possible to discern a few significant tendencies. And these, it turns out, bear remarkable resemblances to developments in Greece. It is known, for instance, that the Śulba geometers of perhaps the fifth or sixth century B.C. discovered the incommensurability of the diagonal of a square with its sides.[8] Having done so, they then devised a means for computing an approximate value of $\sqrt{2}$. Significantly, however, this was perceived as no more than an approximation. This suggests that they were aware that $\sqrt{2}$ is irrational, that is, that its precise value can never be given with a finite string of numerals; and from here it is but a short step to the notion of a number continuum. That is, the mathematician who knows of the existence of irrationals should soon come to see that there are infinitely many numbers between any two consecutive integers. And with this realization comes the notion of infinite divisibility. While we cannot say for certain that the Śulba geometers were consciously aware of infinite divisibility, developments in Indian physics require some source for the notion, and the sophistication of the Śulba school makes it seem the likeliest place to look. The developments to which we refer are the emergence of the curious atomistic doctrines of space and time. Material atomism is quite common in classical Indian philosophy, and it appears to have been maintained by Sāṁkhya, Nyāya, and Sarvāstivāda. For these schools the *paramāṇu* is the ultimate atomic component of all material entities. While it is itself imperceptible, this *paramāṇu* or ultimate atom is the material cause of all sensible objects. It is said to be dimensionless, partless, and indivisible, so that we may say that its size constitutes a spatial minim.[9] In certain respects, however, the *paramāṇu* must be considered infinitesimal, that is, as having some of the properties of a geometrical point. Thus the atomic size of the *paramāṇu* is not properly additive: We should expect the size of the simplest atomic compounds to be a function of two factors—number of component atoms and atomic size—but only the first factor, number, is in fact involved in computing atomic size.[10] This is to say that the measure of a dyadic compound is not twice the size of the constituent *paramāṇu*, but is rather a size which is independently assigned to the dyad. Thus while the idea of an atomic size of the *paramāṇu* suggests a doctrine of spatial minims, the doctrine that this size is nonadditive suggests a conception of a truly dimensionless atom, that is, a point.

Similar tendencies can be seen in some of the classical Indian theories of time. Certainly the Sāṁkhya theory of time must be considered at least quasi-atomistic; the duration required for a physical atom to move its own measure of space is said to be a *kṣana*, or atomic unit of time. And in Abhidharma we find an explicit temporal atomism, based on the notion of *kṣana* as the atomic duration of a *dharma* or atomic occurrence. Here we also see a concern with the problem of divisibility and indivisibility. The *kṣana* is first defined as being of imperceptibly short duration. In order to account for the processes which must occur during the lifetime of a *dharma*, however, the *kṣana* is divided into three constituent phases: arising, standing, and ceasing-to-be. The process of subdivision is then repeated, so that each phase of the *kṣana* itself consists of three subphases, giving in all nine subphases. But here the process of division ends, the subphases being considered partless and indivisible, that is, temporal minims. Thus the subphase can be considered a true atom of time, since it exists outside the flow of time, in the manner of Whitehead's epochs.[11]

The natures of these atomisms in pre-Mādhyamika Indian thought have two important implications. First, they imply acceptance of the principle of discontinuity as it applies to our notions of space and time. This is just what it means to speak of minims of space (*paramāṇu*) and time (*kṣana* subphase). That there can be a least possible length and a least possible duration means that space and time are not continuous but rather discontinuous—for example, time does not flow like an electric clock, but rather it jumps like a hand-wound clock. This is an inescapable consequence of saying that the *paramāṇu* is of definite but indivisible extension, and that the *kṣana* subphase is of definite but indivisible duration.

The second implication of these atomisms is that their proponents implicitly accepted the notion of spatiotemporal continuity. It is one thing to say that the atoms of space or time are indivisible and partless; it is quite another to say that they are dimensionless and nonadditive. The former assertion might be seen as a counter to the argument of the opponent of atomism that since a physical atom is of definite extension, it must itself be divisible and so consist of parts. To this the atomist replies by arbitrarily establishing the measure of the atom as the least possible extension. But the second assertion, that the atom is dimensionless and nonadditive, goes too far. It implicitly accepts the opponent's thesis of infinite divisibility. The property of nonadditiveness properly applies only to true geometrical points on a line. And with this notion comes as well the idea that between any two points on a line there are an infinite number of points; that is, the line consists of an infinite number of infinitesimal points. This notion is, of course, suggested by the discovery of the irrationality of $\sqrt{2}$. Thus we are led to suppose that as with the Pythagoreans, so also in India, the discovery of irrationals led to an atomic doctrine that treated space and time as, in some respects, discontinuous and, in other respects, continuous.

Our aim is to show that some of Nāgārjuna's arguments against motion, like Zeno's Paradoxes, exploit the atomist's assumptions about continuity and discontinuity of space and time. Before we turn to the direct examination of these arguments, however, we must perform one brief final task—we must indicate the point of Nāgārjuna's dialectical refutation of motion. I think we may safely say that Nāgārjuna's chief task in MMK is to provide a philosophical rationale for the notion of *śūnyatā* or "emptiness," which is the key term in the Prajñāpāramitā Sūtras, the earliest Mahāyāna literature. What this comes to is that he must show that all existents are "empty" or devoid of self-existence. He must perform this task in such a way, however, as neither to propound nihilism (which is considered a heresy by Buddhists) nor to generate class paradoxes. To this end Nāgārjuna constructs a dialectic which he considers capable of reducing the metaphysical theories of his opponents (chiefly Sarvāstivāda, Sāṁkhya, and Nyāya) either to contradiction or to a conclusion which is unacceptable to the opponent. Unlike Zeno, however, Nāgārjuna is not refuting the theories of his opponents simply as a negative proof of his own thesis: Nāgārjuna has no thesis to defend—at least not at the object-level of analysis where metaphysical theories compete with one another. Instead his dialectic constitutes a meta-level critique of all the metaphysical theses expounded by his contemporaries. One of Nāgārjuna's chief techniques is to exploit the hypostatization or reification which invariably accompanies metaphysical speculation. This is to say that he is arguing against a strict correspondence theory of truth and is in favor of a theory of meaning, which takes into account such things as coherence and pragmatic and contextual considerations. We may thus say that Nāgārjuna seeks to demonstrate the impossibility of constructing a rational speculative metaphysics.

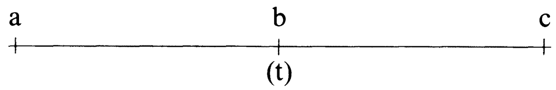
As one step in this demonstration, MMK II seeks to show the nonviability of any account of motion which makes absolute distinctions or which assumes a correlation between the terms of the analysis and reals, that is, any analysis which is not tied to a specific context or purpose but is propounded as being universally valid. Thus once again Nāgārjuna differs from Zeno—here, in that he is not arguing against the possibility of real motion (indeed he argues against rest as well), but only against the possibility of our giving any coherent, universally valid account of motion. To this end he employs two different types of argument: (a) "conceptual" arguments, which exhibit the absurd consequences of any attempt at mapping meaning structures onto an extralinguistic reality; these exploit such things as the substance-attribute relationship, designation and predication; (b) "mathematical" arguments, which exploit the anomalies which arise when we presuppose continuous or discontinuous time and/or space. Arguments of type (a) have already received considerable attention from scholars of Mādhyamika; thus the bulk of the remainder of this article will focus on arguments which we feel belong in category (b).

It is MMK II:1 to which Kajiyama refers when he calls Nāgārjuna's arguments "Zeno-like." And indeed there is a clear resemblance between this and Zeno's Arrow Paradox.

Gataṁ na gamyate tāvadagataṁ naiva gamyate
gatāgatavinirmuktaṁ gamyamānaṁ na gamyate

The gone-to is not gone to, nor is the not-yet-gone-to;
In the absence of the gone-to and the not-yet-gone-to, present-being-gone-to is not gone to.

The model which is under scrutiny here is that which takes both time and space to be continuous, that is, infinitely divisible. The argument focuses explicitly on infinitely divisible space, but infinitely divisible time must be taken as a suppressed premise if the argument is to succeed. Suppose a point moving along a line a–c such that at time (t) the point is at b:



We may then ask, Where does this motion take place? Now clearly present motion is not taking place in the segment already traversed, a–b. Equally clearly, however, present motion is not taking place in the segment not yet traversed, b–c. Thus the going is not occurring in either the gone-to or in the not-yet-gone-to. But for any (t), the length of the line is exhausted by (a–b) + (b–c). That is, apart from the gone-to and the not-yet-gone-to, there is no place where present-being-gone-to occurs. Therefore nowhere is present motion taking place.

Our interpretation is confirmed by Candrakīrti's comments in the *Prasannapadā*:

[The opponent claims:] The place which is covered by the foot should be the location of present-being-gone-to. This is not the case, however, since the feet are of the nature of an aggregate of infinitesimal atoms (*paramāṇu*). The place before the infinitesimal atom at the tip of the toe is the locus of the gone-to. And the place beyond the atom at the end of the heel is the locus of the not-yet-gone-to. And apart from this infinitesimal atom there is no foot.[12]

There are two problems involved in making sense of this passage. The first is that we must assume the goer to be going backwards! This is easily remedied, however, by the convenient device of scribal error. Thus if we assume that an -a- has been dropped between *tasya* and *gate* at lines 21–22, and then inserted between *tasya* and *gate* of line 22,[13] our goer will be moving forward once again. The second problem stems from the fact that for the argument to succeed we must assume that a foot consisting of a single atom is being considered. This does not constitute a serious objection, however, since the analysis may then be applied to any geometrical point along the length of a real foot—it is for this reason that Candrakīrti begins the argument by asserting that our feet are just aggregates of *paramāṇu*. Once these two problem are resolved, it becomes clear that Candrakīrti's interpretation of MMK II:1 involves the explicit assumption of infinitely divisible space and the implicit assumption of infinitely divisible time.

In MMK II:2 Nāgārjuna's opponent introduces the notion of activity or process:

Ceṣṭā yatra gatistatra gamyamāne ca sā yataḥ
Na gate nāgate ceṣṭā gamyamāne gatistataḥ

When there is movement there is the activity of going, and that is in present-being-gone-to;
The movement not being in the gone-to nor in the not-yet-gone-to, the activity of going is in the present-being-gone-to.

This notion of an activity of going, which takes place in present-being-gone-to, requires minimally that we posit an extended present. This is required since only on the supposition of an extended or 'fat' present can we ascribe activity to a present moment of going. Thus the opponent is seeking to overcome the objections against motion which were raised in II:1, which involved the supposition of infinitely divisible time. The opponent's thesis appears to be neutral with respect to space however; it seems to be reconcilable with either a continuous or a discontinuous theory of space.

A textual ambiguity in II:3 has important consequences. Where Vaidya has *dvigamanam*[14] (double going), Teramoto has *hyagamanam*[15] (since a nongoing), and May has *vigamanam*[16] (nongoing). Vaidya's reading seems somewhat more likely, since "double going" is supported by the argument of Candrakīrti's commentary. However both readings yield an interpretation which is consistent with our assumption that in II:3 Nāgārjuna will seek to refute the case of motion in discontinuous time. Thus on Vaidya's reading II:3 is:

Gamyamānasya gamanaṁ kathaṁ nāmopapatsyate
gamyamāne dvigamanaṁ yadā naivopapadyate

How will there occur a going of present-being-gone-to
When there never obtains a double going of present-being-gone-to?

On this reading the argument is against the model of motion which assumes that both time and space are discontinuous; thus it parallels in function Zeno's Paradox of the Stadium. Suppose that time is constituted of indivisible minims of duration d, and space is constituted of indivisible minims of length s. Now suppose three adjacent minims of space, A, B, and C, and suppose that an object of length 1s at time t_0 occupies A and at time t_1 occupies C, such that the interval t_0-t_1 is 1d. Now since the object has been displaced two minims of space, that is, 2s, this means that its displacement velocity is $v = 2s/d$. For the object to go from A to C, however, it is clearly necessary that it traverse B, and so the question naturally arises, When did the object occupy minim B? Since displacement A-B is 1s, by our formula we conclude that the object occupied B at $t_0 + \frac{1}{2}d$. This result is clearly impossible, however, since d is posited as an indivisible unit of time. And yet the notion that the object went from A to C without traversing B is unacceptable. In order to reconcile theory with fact, we might posit an imaginary going whereby the object goes from A through B to C, alongside the orthodox interpretation whereby the object goes directly from A to C without traversing B. This model requires two separate goings, however, and that is clearly absurd. Thus we must conclude that there is no going of present-being-gone-to, since the requisite notion of an extended present leads to absurdity.

If we accept Teramoto's or May's reading, then II:3 becomes:

Then how will there obtain a going of present-being-gone-to,
Since there never obtains a nongoing of present-being-gone-to?

This may be taken as an argument against the model of motion which presupposes discontinuous time but a spatial continuum. Suppose that time is constituted of indivisible minims of duration d. Now suppose that a point is moving along a line a-c at such a rate that at t_0 the point is at a, and at $t_1 = t_0 + 1d$, the point is at c. Now by the same argument which we used on the first reading of II:3, for any point b lying between a and c, b is never passed by the moving point, since motion from a to b would involve a duration less than d, which is impossible. Thus what we must suppose is that for some definite duration d, the point *rests* at a, and for some definite duration d, the point *rests* at c. The whole point of the supposition at II:2 was to introduce the notion of activity, however. Now it seems that this supposition leads to a consequential non-going, which is not only counterintuitive but also clearly contrary to what the opponent sought when he presupposed an extended present. While the principles of cinematography afford a good heuristic model of a world in which time is discontinuous and space continuous, we do not recommend them to anyone interested in explaining present motion through a spatial continuum.

MMK II:4–6 continues the argument against the opponent of II:2. Verse 4 is a good example of Nāgārjuna's "conceptual" arguments against motion, which frequently exploit the realistic assumptions behind the Abhidharma *lakṣaṇa* doctrine of designation:

Gamyamānasya gamanaṁ yasya tasya prasajyate
ṛte gatergamyamanaṁ gamyamānaṁ hi gamyate.

If there is a going of present-being-gone-to, from this it follows,
That present-being-gone-to is devoid of the activity of going (gati), since present-being-gone-to is being gone to.

Candrakīrti's commentary, with its use of terms borrowed from the grammarians, brings out the linguistic nature of the argument:

The thesis is that there is going (*gamana*) through the designation of present-being-gone-to; what obtains the action of going (*gamikriyā*), which is an existent attribute, from present-being-gone-to, which is a non-existent term devoid of the action of going; of that there follows the thesis that present-being-gone-to is without the activity of going (*gati*), [since] going (*gamana*) would be devoid of the activity of going (*gati*). Wherefore of this, "Present-being-gone-to is being gone to" [is said]. The word "*hi*" means "because." Therefore because of the saying that present-being-gone-to, though devoid of the activity of going (*gati*), is truly gone to, here the action (*kriyā*) [of going] is employed, and from this it follows that going (*gamana*) is devoid of the activity of going (*gati*).[17]

In order for us to understand this, it is necessary that we back up for a moment and look at Candrakīrti's comments on II:2. There he has the opponent elaborate his supposition with the following remarks: "Where *gati* is obtained, that is present-being-gone-to, and that is known from the action of going. It is for just this reason that present-being-gone-to is said to be gone-to. The one is for the purpose of knowledge (*jñānārtha*), and the other is for the purpose of arriving at another place (*deśāntarasamprāptyartha*)."[18] The opponent's thesis is that movement or the process of going is to be found in the moment of present-being-gone-to; but since the latter is not an abiding feature of our world, but rather just a convenient fiction or conceptual fiction, there must be available some mark or characteristic whereby it is known or singled out. This mark is the action of going (*gamikriyā*). The referent of this attribute is the real process of going, namely, *gati*, the activity of going. The term *gamana*, 'going', is now introduced in order to signify the product of the assertion that present-being-gone-to is being gone to, namely, the going whereby present-being-gone-to is supposedly being gone-to.

Nāgārjuna's argument is that by speaking of a *going* of present-being-gone-to, we forfeit the right to speak of an *activity of going* of present-being-gone-to. Candrakīrti's elaboration of this argument may be put as follows: The object of the opponent is to locate the activity of going in present-being-gone-to, but before this can be done he must first isolate this moment. Since the notion of present-being-gone-to is abstracted from a complex historical occurrence, it is necessary that it be designated through the arbitrary assignment to it of the action of going, that is, we locate the moment of present-being-gone-to by defining it as that wherein the action of going takes place. So far there is nothing objectionable in the opponent's procedure. We run into difficulties, however, when he insists that through this assignment of the action of going to present-being-gone-to, this moment has obtained real going, that is, it is truly gone-to. For in this case *gamikriyā*, ostensibly the *lakṣaṇa* or mark of *gati*, has in fact become the *lakṣaṇa* of *gamana*, the purported going of present-being-gone-to. The attribute action-of-going cannot be used at once to refer to the real activity of going and also to designate the construct present-being-gone-to, if the result of the latter designation is the attribution of going-to this present moment. Either of these two tasks—reference to a real activity of going or designation of the construct present-being-gone-to-with-consequent-going—exhausts the function of the *lakṣaṇa* action-of-going.

Nāgārjuna pushes this point in II:5–6. In verse 5 he notes that the thesis of the opponent leads to two goings—that by which there is present-being-gone-to, and that which is the true going. Since the designation of present-being-gone-to as truly gone to has led to the exhaustion of the *lakṣaṇa* action-of-going in assigning a going whereby the present moment is gone-to, the attribute action-of-going is now incapable of imparting its purported referent, real activity of going (*gati*), to the going (*gamana*) which is assigned to present-being-gone-to. We must now imagine two goings, one by which present-being-gone-to is purportedly gone-to, and another which obtains the real attribute of the action of going and which thus stands for the activity of going. And as Nāgārjuna points out in verse 6, the consequence of this supposition is two goers, since without a goer there can be no going.

To those unfamiliar with Mādhyamika dialectic, the argument of II:4–6 must seem sheer sophistry. Here two things must be borne in mind. First, Nāgārjuna's argument is aimed at a historical opponent, not at a straw man; seen in the light of this historical context, the argument seems somewhat less specious. The thesis that there is a 'fat' temporal present within which motion to an other takes place was held by at least one Abhidharma school, the Pudgalavādins.[19] And the *lakṣaṇa* criterion, whereby only that is a real (that is, a *dharma*) which bears its own *lakṣaṇa* or defining characteristic, was held in common by all the Abhidharma schools. This latter doctrine, when taken in conjunction with the strict correspondence theory of truth which was the common position of early Buddhism, yields precisely the excessively realistic attitude toward language which Nāgārjuna so consistently exploits throughout MMK. In particular, Nāgārjuna is here taking to task the opponent's assumption of the possibility of real definition—the proper manipulation of linguistic symbols gives us insight into the constitutive structures of extralinguistic reality—and with it the assumption of language-reality isomorphism.

Seen in this light, however, the opponent's presuppositions are neither as farfetched nor as alien to our own philosophical concerns as they might have seemed. And this brings us to the second point we should like to make about Nāgārjuna's line of argument in II:4–6: The attack is not against motion per se but against a certain attitude toward language, and so its basic point will have effect wherever noncritical metaphysics is practiced. The argument relies on the fact that the outcome of an analysis depends, among other things, on the purpose behind doing the analysis. Thus the notion of a definitive analysis of motion is inherently self-contradictory. Any account which purports to be such an analysis can be shown to be guilty of hypostatization. When the terms of the analysis—here, in particular *gati* and *gamyamāna*—are taken to refer to reals, they immediately become reified, frozen out of the series of systematic interrelationships which originally gave them, as linguistic items, meaningfulness. This necessitates the notion of a separate apellate 'going' whereby the real going or the real present-being-gone-to is known. This, in turn, gives rise to the problem of the logical interrelations among these various terms. The result is Nāgārjuna's demonstration that the supposition of motion in an extended present leads to paradoxical consequences. The point we wish to make about this demonstration is that its efficacy extends far beyond the limited scope of Pudgalavādin presuppositions. Even more than his and Zeno's "mathematical" arguments, Nāgārjuna's "conceptual" arguments against motion are of greater than merely historical interest.

MMK II:7–11 seeks to further demonstrate the impossibility of motion by focusing on the notion of a goer. In verse 7 Nāgārjuna states the obvious point that there is a goer if there is a going. Verses 8 and 9 then convert this, by means of the conclusion of II:1–6 that no going occurs in the three times, to the consequence that there can be no goer. MMK II:10–11 then utilize essentially the same argument as verses 4–5, but here apply it to the notion of a goer:

Pakṣo gantā gacchatīti yasya tasya prasajyate
gamanena vinā gantā gantur-gamanamicchataḥ.
Gamane dve prasajyate gantā yadyuta gacchati
ganteti cocyate yena gantā san yacca gacchati

The thesis is that the goer goes: from this it follows
That there is a goer without a going, having obtained a going from a goer.
Two goings follow if the goer goes:
That by which "the goer" is designated, and the real goer who goes.

Here again we see that the assumption of language-reality isomorphism leads to paradoxical consequences; in this case the analysis of the notion of a goer leads to two goings, one on the side of language, the other on the side of reality.

MMK II:12–13 allows two divergent interpretations: one takes it to be an argument of the "mathematical" type, the other to be an argument of the "conceptual" type. The verses are as follows:

Gate nārabhyate gantuṁ gantaṁ nārabhyate 'gate
Nārabhyate gamyamāne gantumārabhyate kuha.

Na pūrvaṁ gamanārambhād gamyamānaṁ na vā gataṁ
yatrārabhyeta gamanamagate gamanaṁ kutaḥ.

Going is not commenced at the gone-to, nor is going commenced in the not-yet-gone-to;
It is not begun in present-being-gone-to; where, then, is going commenced?

Present-being-gone-to does not exist prior to the commencement of going, nor is there a gone-to
Where going should begin; how can there be a going in the not-yet-gone-to?

The "mathematical" interpretation of this argument assumes infinitely divisible time, or a temporal continuum. No special assumptions about the nature of space are required, so that space may be taken as either continuous or discontinuous. The argument may thus be taken to correspond in function to either Zeno's Arrow Paradox or to the Paradox of Achilles and the Tortoise. Assume an individual, Devadatta, who during the interval t_0-t_1 is standing at a given location, and at some time during the interval t_1-t_2 leaves that location. Then assume that there is some time t_x contained in the interval t_1-t_2, subsequent to which Devadatta is going. We may now ask when Devadatta commenced to go. The interval t_o-t_x exhaustively describes the duration of Devadatta's not-going. And the interval t_x-t_2 exhaustively describes the duration of Devadatta's going for the period that concerns us. Then since $(t_o\text{-}t_x) + (t_x\text{-}t_2)$ covers the entire duration of the analysis, we must conclude that at no time does Devadatta actually commence to go, that is, at no time does the activity of commencing to go take place. Similarly, where i is an infinitesimal increment in duration (that is, a *kṣana* subphase), then for any n, $(t_1 + n \cdot i) < t_x$ and $(t_2 - n \cdot i) > t_x$ Therefore at no time does the commencement of going take place.

The "conceptual" interpretation of this argument goes as follows: The gone-to, the not-yet-gone-to, and present-being-gone-to, as temporal moments, are not naturally occurring existents, but rather conventional entities defined in relation to going. It is therefore impossible to designate these three moments prior to the commencement of going. It is impossible to speak of going actually taking place, however, without this division of the temporal stream into the three moments of gone-to, etc. In other words, a necessary condition of our recognizing the commencement of going is our being in a position to speak of a time where going has ceased, a time where going is presently taking place, etc. If we are to succeed in designating the commencement of going, it must take place in one of these three moments—and, of course, the not-yet-gone-to may be exluded from our considerations as a possible locus of the commencement of going, since by definition no going may take place in it. Thus the commencement of going must take place either in the gone-to or in present-being-gone-to. This is impossible, however, since neither of these moments may be designated prior to the commencement of going.

Candrakīrti's commentary on II:13 appears to support this interpretation: "If Devadatta is standing, having stopped here, then he does not commence going. Of him prior to the beginning of going there is no present-being-gone-to having its origin in time, nor is there a gone-to where going should be begun. Therefore from the non-existence of gone-to and present-being-gone-to, there is no beginning of going."[20]

A moment's reflection will show, however, that this interpretation is not substantially different from the "mathematical" interpretation of the argument, particularly the second version, which made use of infinitesimal increments of duration. Indeed on this interpretation the argument seems specious unless we make the additional assumption that its target includes a "knife-edge" picture of time. Thus if one assumes that time is continuous and infinitely divisible, then at the instant (that is, time-point) at which going actually commences, there is in fact no real motion, since this is just the dimensionless dividing-line between the period of rest and the period of motion. And no matter how many infinitesimal increments one adds to the period of rest after it has supposedly terminated, the same situation will prevail. Moreover, as long as one is unable to locate real motion, one will likewise be unable to discern a gone-to and present-being-gone-to. This means, however, that we will never succeed in designating a commencement of going. Nāgārjuna summarizes the results of II:12–13 in verse 14:

Gataṁ kiṁ gamyamānaṁ kimagataṁ kiṁ vikalpyate
adṛśyamāna ārambhe gamanasyaiva sarvathā.

The gone-to, present-being-gone-to, the not-yet-gone-to, all are mentally fabricated,
The beginning of going not being seen in any way.

In the remaining verses of Chapter II (15–25) Nāgārjuna continues his task of refuting motion by defeating various formulations designed to show how real motion is to be analyzed. Thus, for example, in II:15 the opponent argues for the existence of motion from the existence of rest; that is, since the two notions are relative, if the one has real reference, the other must also. In particular we may speak of a goer ceasing to go. As Nāgārjuna shows in II:15–17, however, the designation of this abiding goer is even more difficult than the designation of a goer who actually goes. There are also arguments concerning the relationship between goer and activity of going, and the relationship between goer and that which is to be gone-to. None of these introduces any new style of argumentation, however; all seem to be variations on objections already raised. In particular, none of the arguments presented in these verses is susceptible to a "mathematical" interpretation. Thus we shall bring our analysis of MMK II to a close here, merely noting in passing that where Zeno has four Paradoxes, one designed to refute each permutation of the ramified

Pythagorean spatiotemporal analysis, we have succeeded in uncovering only three such arguments in Nāgārjuna. The first (II:1) covers the case of infinitely divisible space and infinitely divisible time; the third (II:12–13) deals with infinitely divisible time, and thus covers the two cases of discontinuous space and infinitely divisible time, and continuous or infinitely divisible space and infinitely divisible time (already covered by II:1). The second "mathematical" argument (II:3), depending on how one reads it, covers either discontinuous space and discontinuous time (Vaidya), or continuous, infinitely divisible space and discontinuous time (Teramoto, May). Thus depending on which text of II:3 is rejected, the corresponding permutation of the four possible analyses will not be covered by Nāgārjuna's arguments.

The natural philosophies against which Zeno and Nāgārjuna argue are surprisingly similar. It seems likely that in each case the account in question began as an atomism, maintaining that the universe is additive and that it is composed of some sort of minims or atoms; we can then suppose that each of these theories was severely shaken by the discovery of $\sqrt{2}$ and the incommensurability of the hypotenuse of a unit right triangle with its side, which prove the impossibility of proper minims. However the result of this discovery was, in each case, not the abandonment of atomism, but an ill-fated attempt to reconcile that atomism with the new mathematical knowledge, an attempt which resulted in great confusion and inconsistency.

Zeno and Nāgārjuna attack these muddled systems for similar reasons. Neither is constructing a system or defending a thesis of his own; each is, instead, attacking his opponents' positions to provide indirect proof of an established doctrine. The doctrines defended are, however, completely different in kind. Zeno argues against pluralism to support the monism of his teacher Parmenides, a theory of the same type as that being rejected. Nāgārjuna, on the other hand, attacks pluralism, among other theories, to support the doctrine of emptiness, a doctrine of a higher logical order than those which he refutes. There is a further difference between the two philosophers, in that, unlike Zeno, Nāgārjuna designs his refutations as much to elucidate his chosen doctrine as to defend it: In providing a philosophical rationale for "emptiness" he is exhibiting the true import of this term, which occurs essentially undefined in the Prajñāpāramitā literature. In showing *why* all *dharmas* are empty, Nāgārjuna gives the first truly formal account of the meaning of this doctrine.

There are also important similarities between the two philosophers' styles of argument. Both, as we have seen, are given to the use of indirect proof. Both make use of a "mathematical" style of argument which accepts the opponent's premises and demonstrates that they entail either absurdities or consequences unacceptable to the opponent. However, Nāgārjuna also makes use of a very different sort of argument—one which approaches the problem in question from a meta-level, showing the problem as one of reification, arising from the opponent's attempt to project his analysis out onto some extralinguistic "reality," and to make the terms of this analysis correspond to independent entities in that "reality." There are other differences as well. Zeno is far more formal and systematic in his arguments than is Nāgārjuna in his "mathematical" arguments; Zeno constructs Paradoxes to cover all four possible cases of spatiotemporal continuity and/or discontinuity, whereas Nāgārjuna has only three arguments, and these tend to overlap. On the other hand, Nāgārjuna seems more clearly aware of the nature of his opponents' fallacy, the confusion of mathematical analysis with physical occurrence and of mathematical fictions or conventions with physical entities.

By means of their various arguments concerning motion, both Zeno and Nāgārjuna reach the conclusion that no intelligible account of motion is possible. However, the two proceed from this point of agreement in quite different directions. Zeno concludes that since no intelligible account of motion can be given, and since the unintelligible cannot exist, therefore motion itself is impossible, and Being must be unmoving. This supports Parmenides' doctrine that Being is one and unchanging. Nāgārjuna concludes instead that it is impossible to give an intelligible account of motion because to do so is to attempt to make a description or analysis designed to cope with a certain limited practical problem apply far beyond its sphere of competence. This in turn supports the thesis that metaphysics is a fundamentally misguided undertaking. One could only tie everything up into one neat bundle if there were some single extralinguistic reality, "the world," out there standing as guarantor of the veracity of one's account. The nature of "reality," which is just our *experience* of a *constructed* world, is determined by the nature of the language in which it is described—and that varies according to the task at hand. For this reason any rational speculative metaphysics is impossible.

As has been noted by others, the two philosophers' treatments of motion are remarkably similar, despite their great separation in time, place and culture. What differences there are between the two can largely be accounted for by the differing purposes of these accounts.

NOTES

1. Kajiyama Yūichi, *Kū no Ronri* (Tokyo: Kadokawa Shoten, 1970), p. 89.
2. T. R. V. Murti, *The Central Philosophy of Buddhism* (London: George Allen and Unwin, 1960), pp. 178, 183–184.
3. Robert S. Brumbaugh, *The Philosophers of Greece* (New York: Thomas T. Crowell Co., 1964), pp. 57–67.
4. G. S. Kirk and J. E. Raven, *The PreSocratic Philosophers* (Cambridge: Cambridge University Press, 1969), pp. 292–3. The English translations follow Gaye.
5. Kirk and Raven, p. 294.
6. Kirk and Raven, p. 294.
7. Kirk and Raven, pp. 295–296.

8. Bibhitibusan Datta, *The Science of the Śulba* (Calcutta: University of Calcutta, 1932), pp. 195–202.

9. The use of the notion of atomic size in the Sāṁkhya theory of time involves the conception of a spatial minim, or a finite indivisible length. Confer below.

10. Surendranath Dasgupta, *A History of Indian Philosophy* (Cambridge: Cambridge University Press, 1922), vol. 1, pp. 314–315.

11. Both Sāṁkhya and Abhidharma hold that time, unlike space, is not an ultimate constituent of reality. They appear to maintain, like Whitehead, that our notion of temporal flow is derivative and secondary, a product of the occurrence of atomic occasions. This is the basis for Nāgārjuna's rejection of the Abhidharma theory in MMK XIX:6. But the ultimate unreality of time does not detract from the significance of the kṣana theory for our considerations.

12. Yamaguchi Susumu, trans., *Gesshōzō Chūronshaku* (Tokyo: Kōbundō, 1951).

13. *Mādhyamakaśāstra of Nāgārjuna*, P.L. Vaidya, ed. (Dharbanga: Mithila Institute, India, 1960), p. 33.

14. Vaidya, p. 34.

15. Teramoto Enga, trans. and ed., *Chūronmuiso* (Tokyo: Daitō Shuppansha, 1938), p. 42.

16. Candrakirti, *Prasannapadā Madhyamakavṛtti*, Jacques May, trans. (Paris: Adrien-Maisonneuve, 1959), p. 55, n. 19.

17. Vaidya, p. 34. Not only is Yamaguchi's translation of this passage (p. 149) incomprehensible, it also ignores the grammar of the original.

18. Vaidya, p. 34; Yamaguchi, pp. 145–146.

19. Yamaguchi, p. 146.

20. Vaidya, p. 37.